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A joint trade and mode chain choice modelling framework with application to Italy

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Abstract

The needs of governments of European countries in the area of goods mobility include prediction, in base year and in policy scenarios, of monetary trade flows as well as weight transport flows by mode chain. The models and datasets that can be used are reviewed. For trade flows, the structural gravity model, which is standard for international trade analysis. For transport flows by mode chain, the Logit model. Joint non-survey calibration of the gravity and Logit models is feasible on the basis of Eurostat transport data. Regional Input-Output tables as well as travel time, cost and route models provide the needed inputs. Aggregation of flows considers the sectors of solid bulk, liquid bulk, and container and general cargo. Price indices needed to convert monetary flows into weight flows in the prediction stage are obtained from Eurostat data. The proposed approach is applied to Italy.

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1. Introduction

The development of a Decision Support System (DSS) in the area of goods mobility for governments of European countries is the subject of the present paper. In particular, the focus is on the prediction, in base year and in counterfactual scenarios, of trade flows (EUR/year) and transport flows by mode chain (tonnes/year).

The interaction between economy and goods mobility occurs in two ways. Transport demand is derived from production and consumption activities, the direct link. Buy and sell markets are dependent on accessibility, the inverse link. Any DSS which governments may use to evaluate policies and projects, should give proper accounts of the two links. To this aim, the approach based on Multi-Regional Input-Output (MRIO) with elastic trade coefficients has been proposed (Zhao and Kockelman, 2004; Cascetta et al., 2013).

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The choice of the mode chain is usually tackled in a distinct and subsequent modelling step. Traditionally, Logit models are used (de Jong, 2014). A noteworthy example is the chain choice model at European level in Jensen et al. (2019). Transferability from available models of the alternative specific constants is incorrect, which makes the calibration of models specific to the geographic context inescapable.

In this paper, a joint trade and mode chain choice modelling framework is proposed. For trade, the structural gravity model is used (Yotov et al., 2016). This is standard today for international trade analysis, because of its foundation in economic theory. Logit is used for the choice of the mode chain.

The combination of the gravity model and the mode chain choice model makes forecasting of a wide range of substitution effects, from changes in accessibility, possible. These include changing zone of intra-national trade, changing country of international trade, changing mode chain.

The structural gravity model requires inputs from regional Input-Output (IO) tables. Since IO tables are available at country level, regionalisation, with reference to the local geographic level selected for the analysis, is needed.

For joint calibration of the trade and the chain choice models, a non-survey methodology, which uses only available data from international statistic infrastructure, is proposed. This circumvents the need for ad-hoc surveys, which are complex and costly, due to the multiplicity of stakeholders and goods.

In this study, particular attention is assigned to aggregation issues, since classification of goods is specific of dataset and model.

The paper has the following organization. Section 2 presents the models of the framework, with mathematical details, as well as specification of the datasets used to provide the inputs that are needed. Section 3 explains how the models can be used for transport policy analysis. Section 4 concludes.

2. Modelling framework

2.1. Zoning

Italy is the country of interest. For Italy, the zoning structure is defined at NUTS3 level, the one of former provinces. Therefore, inside Italy, there are 107 zones. Outside Italy, 68 zones are defined, at country level for the European continent and the Mediterranean area, and continental or sub-continental level for the rest of the world.

2.2. Sectors

Three sectors are considered based on load type (the same choice made by the European chain choice model in Jensen et al., 2019): dry bulk, liquid bulk, other goods. Other goods include container and general cargo. General cargo, also referred to as break-bulk cargo, includes unitised cargo (e.g. pallets) and loose cargo (e.g. machinery and vehicles). For ease of notation, in each of the models in the sequel, the sector index is omitted.

2.3. Gravity trade models

The structural gravity model of trade is founded in economic theory (Yotov et al., 2016). Consumers choose goods from different production zones. Consumer preferences are assumed to be homothetic, identical across consumption zones, and given by a constant elasticity of substitution (CES) utility function for each zone. Consumers maximize this utility under a budget constraint. Market is cleared, i.e. the production of each zone is traded with consumption zones. Consumers pay a price equal to the factory-gate price times a trade cost markup associated with transport. The model provides trade flows at the aggregate level and the level of individual sectors.

Let J_1 and J_2 be the sets of, respectively, Italian and foreign zones. Only trade involving at least one Italian zone as production or consumption zone is modelled. Trade from foreign to foreign zone is not modelled.

The trade flow, in EUR/year, between production zone i and consumption zone j (from Italian zone to a different Italian zone, from Italian to foreign zone, from foreign to Italian zone) is:

$$X_{ij} = \frac{Y_i E_j}{Y} \left(\frac{\tau_{ij}}{\psi_i \omega_j} \right)^{1-\sigma}, (i \in J_1, j \in J_1/i) \cup (i \in J_1, j \in J_2) \cup (i \in J_2, j \in J_1) \quad (1)$$

where:

Y_i regional or foreign export, i.e. production sold to Italian or foreign zones,

E_j regional or foreign import, i.e. expenditure of Italian or foreign zones,

Y total regional or foreign export across all production zones,

$\sigma > 1$ elasticity of substitution,

τ_{ij} bilateral trade cost markup associated with transport,

ψ_i regional or foreign outward multilateral resistance, proxy for ease of market access of production zone,

ω_j regional or foreign inward multilateral resistance, proxy for ease of market access of consumption zone.

Multilateral resistances satisfy the nonlinear equations:

$$(\psi_i)^{1-\sigma} = \sum_j \frac{E_j}{Y} \left(\frac{\tau_{ij}}{\omega_j} \right)^{1-\sigma}, \quad (2)$$

$$(\omega_j)^{1-\sigma} = \sum_i \frac{Y_i}{Y} \left(\frac{\tau_{ij}}{\psi_i} \right)^{1-\sigma}, \quad (3)$$

In Eq. (2), summation with respect to j in the r.h.s. is extended to $J_1/i, J_2$ or J_1 depending on the production-consumption pair considered in (1). In Eq. (3), summation with respect to i in the r.h.s. is extended to J_1, J_1 or J_2 depending on the production-consumption pair considered in (1).

By virtue of Eqs (2) and (3), the gravity model is doubly constrained: the sum of modelled trade flows from a production zone i to consumption zones equal the production of zone i , the sum of modelled trade flows from production zones to a consumption zone j equals the consumption of zone j . This holds separately for regional and foreign trade.

2.4. Mode chain choice models

Mode chain alternatives include all-road, road (for access/egress to/from railway terminals) and rail, road (for access/egress to/from ports) and maritime transport. RoRo has attracted the interest of policy makers. Air is relevant to the other goods sector only. Choice of mode chain is modelled by a Logit which provides, for the pair of production and consumption zones (ij), the probability of choosing alternative n :

$$P_{ij,n} = \frac{\exp(-C_{ij,n} + ASC_n)}{\sum_{n' \in N} \exp(-C_{ij,n'} + ASC_{n'})}, n \in N \quad (4)$$

where:

N set of mode chain alternatives,

$C_{ij,n}$ transport generalised cost,

ASC_n alternative specific constant.

The transport generalised cost is:

$$C_{ij,n} = -(\beta_t t_{ij,n} + \beta_c c_{ij,n}), n \in N \quad (5)$$

where:

β_t, β_c estimation coefficients,

$t_{ij,n}$ travel time,

$c_{ij,n}$ monetary shipment cost per tonne.

The bilateral trade cost markup associated with transport is the expectation of the monetary shipment cost per tonne c_{ij} which is given by:

$$c_{ij} = \sum_{n \in N} P_{ij,n} c_{ij,n} \quad (6)$$

2.5. Regional Input-Output tables

The yearly values of goods imported/exported between Italian provinces and with foreign zones Y_i and E_j are estimated using Italy’s national Input-Output (IO) tables from the Istituto Nazionale di Statistica (Istat) dataset, and international trade data from the United Nations Conference on Trade and Development (UNCTAD) dataset.

The supply tables represent the supply of goods at basic price, and the trade margins, transport and taxes minus contributions; the resources of supply are domestic production and import. The use tables show the consumption, at both basic price and purchase price, of imported goods and total goods, separately, by industries (for their production, referred as intermediate uses hereafter) and by final users (e.g. families and public administrations, hereafter referred as final uses) as well as for export. CPA2.1 is used to classify goods, NACE rev.2 to classify industries. Account balances per commodity and per industry between supply and use are shown in Fig. 1.

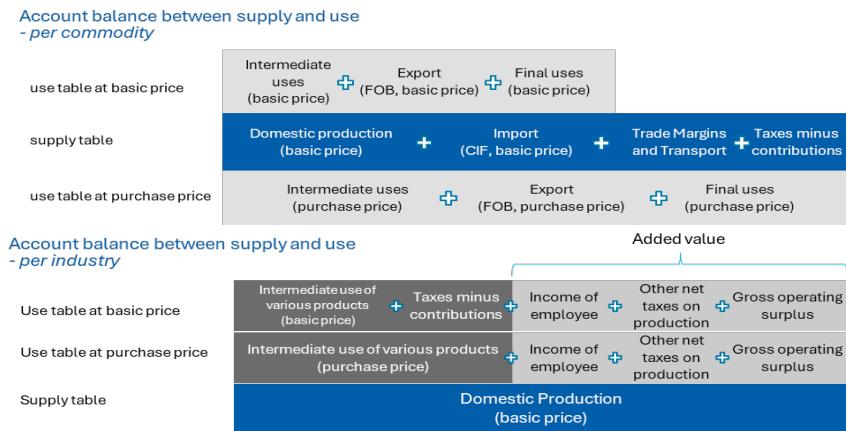


Fig.1. Account balances between supply table and use tables

Regional IO tables (here at province level) should demonstrate, like the national ones, the local supply, the consumption of commodities that are locally supplied and imported from areas outside; the latter part is divided again into two groups: domestic products and foreign products. Starting from the national supply and use tables, a two-steps local IO table generation process is designed: firstly, we estimate the provincial production as well as the consumption of foreign and non-foreign products; then the consumption of local supplied products is estimated among the consumed non-foreign products. It is fundamental that the local IO tables at the finest zoning level must be generated directly from the national ones, and the supplies and expenditures at upper level are the aggregation of the provincial ones only after having completed the assignment of trade flows between origin zone and destination zone at the finest level. This design aims to avoid any geographic assignment / limitation of exchange flows during the local IO tables generation process: if the national tables are localized firstly at regional level and then provincial level, it will automatically imply a certain part of flows as intra-regional.

At the first step, the localization coefficients are calculated based on the percentage of local employment per industry, population, number of public institutions, value added per inhabitant and taxable income. The Regional Economic Accounts, which contain the value added, are used in an iterative apportionment process to calibrate the supply and use estimated by the localization coefficients. The national supply and use table at basic price are adopted because both the balances per commodity and industry at basic price are necessary for obtaining a local table with balanced supply and use and give the possibility to use the Regional Economic Accounts as constraints.

To determine the provincial consumption of local supplied commodities, the Flegg’s Location Quotients are applied to estimate the self-sufficiency level of each industry in each province. Location quotient (LQ) methods are among the most popular non-survey tools for assessing the degree of regional self-sufficiency. LQ methods consider the specialization of production in the region, the relative size of the activities and of the region (Round, 1978). Flegg’s Location Quotients (FLQ) of Eqs (7) and (8) below was developed and reformulated to overcome the limitation of

Cross-Industry Location Quotients (CILQ) of not taking into account the relative size of the study area, and the limitation of Simple Location Quotients (SLQ) of not considering the relative size of the consumer sector (Flegg, et al., 1995; Flegg and Webber, 1997). The FLQ offers better accuracy in regionalising the national IO table (Flegg and Webber, 1997; Tohmo, 2004; Bonfiglio and Chelli, 2008; Flegg and Tohmo, 2013).

$$q_{ms}^R = \begin{cases} 1, & FLQ_{ms}^R \geq 1 \\ FLQ_{ms}^R, & otherwise \end{cases}, \quad \lambda^* = \left[\log_2 \left(1 + EM^R / EM^N \right) \right]^\delta \quad (7)$$

$$FLQ_{ms}^R = \begin{cases} \lambda^* \frac{EM_m^R / EM_m^N}{EM_s^R / EM_s^N}, & m \neq s \\ \lambda^* \frac{EM_m^R / EM_m^N}{EM^R / EM^N}, & m = s \end{cases} \quad (8)$$

where q_{ms}^R is the degree of self-sufficiency which indicates in area R the share of intermediate demand of sector s for sectoral production m satisfied by local production. $EM_m^R, EM_m^N, EM^R, EM^N$ are, respectively, the number of employees in sector m and the total number of employees at local and national level.

In our case, we need to determine the self-sufficiency of a commodity, not a sectoral production. Consequently, the nominator EM_m^R / EM_m^N in Eq. (8), which represents the dimension of supplier sector m , is replaced by the dimension of the local supply of the commodity r , defined as $\sum_m \frac{PR_{r,m}}{PR_r} l_m^R$, where $PR_{r,m}$ is the production of the commodity r by the industry m , PR_r is the total production of r , both at national level, l_m^R is the localization coefficient of the industry m in the local area R . Then, the degree of self-sufficiency of industry s with respect to commodity r in the region R is denoted by q_{rs}^R .

The conversion of the provincial IO tables from basic price to purchase price is taken place right after. It is assumed that the tax rate of production and consumption are the same at both national and provincial level. For each national and imported commodity, the consumption tax rate at national level is calculated as follows:

$$TC_{r,m} = (EX_{rm} - EX'_{rm}) / EX'_{rm}, \quad m \neq export \quad (9)$$

where EX_{rm} is the expenditure of industry or final user m on the commodity r at purchase price, $EX'_{r,m}$ is the expenditure of industry or final user m on the commodity r at basic price. Then, based on the account balances shown in Fig. 1, the consumption tax of the imported products is deducted from the sum of Trade Margins and Transport and Taxes minus contributions at the production side to obtain the production tax. This tax is then distributed among the different industries according to their production value. The production tax rate at national level is then calculated as:

$$TP_{rs} = (PR_{rs} - PR'_{rs}) / PR'_{rs} \quad (10)$$

Inter-provincial and international import/export values are determined through a provincial account balancing process. It is assumed that each Italian province exports only locally produced commodities to any other zones and can re-export foreign products exclusively to international destinations. For each commodity, the local production is first allocated to meet internal demand for local products within the province; any surplus production is designed for export. The value of international exports from each province is determined in proportion to its share in the total export of all provinces; any remaining part is exported to other provinces. In case where the previously estimated local production is insufficient to satisfy internal demand, the shortfall is compensated by imports from other Italian provinces, resulting in an increase in domestic inter-provincial trade flows. At this stage, the values of international export derived from local IO tables are expressed in Free On Board (FOB) terms, international import values are in Cost Insurance and Freight (CIF) terms, and all international and national export / import are reported at purchase price.

The UNCTAD trade dataset provides information for each commodity classified under the HS17 system, including the FOB value at the country of origin, international transport costs and weight. To ensure consistency with national IO tables, commodities are mapped from HS18 to CPA2.1 via the CN17 classification. Their values are then adjusted

to reflect those reported in the Italian national IO tables, while preserving the original ratio of transport cost and FOB cost per kilogram. This procedure yields the CIF value for the international export of Italy, harmonized with the export flows reported in the IO tables. Accordingly, the conversion factors from FOB to CIF prices for the international export derived from IO tables in the previous step are calculated, applied to international exports at provincial level. The harmonized UNCTAD export / import flows are aggregated according to the zoning system and used to determine the share of each foreign zone in Italy's international trade. Absolute trade values are then computed by applying these proportions to the import/export value from national IO tables. Notably, values of gas / petroleum related liquid commodities are excluded, as these products fall outside the scope of the present study.

Finally, the international and national export / import of Italian provinces (estimated from IO tables), the export / import of Italy with foreign zones (derived from UNCTAD data and aligned with IO tables) are categorized into three freight types (liquid bulk, solid bulk, other goods) based on the CPA2.1-CN17/07/08-NST/R correspondence and the Swedish national freight transport system SAMGODS (Vierth and Lindgren, 2017) with our modifications.

2.6. Supply and route models

Network models, with associated travel times (accounting for transshipments) and monetary costs, are implemented using Transcad transportation planning software.

The initial road network is based on EuroGeographics data. The countries manually integrated into the network are UK, Bosnia and Herzegovina, Montenegro, Albania, Turkey, Norway, Sweden, Finland, Belarus and Russia. The final road network model comprises over 460 000 links, covering approximately 240 000 kilometres (Fig. 2).

The initial railway network also is based on EuroGeographics data. National networks have been connected at country borders. Additionally, manual connections have been established for UK, Bosnia and Herzegovina, Montenegro, Albania, Turkey, Norway, Sweden, Finland, Belarus and Russia. Overall, the railway network model comprises over 100 000 links covering approximately 210 000 kilometres (Fig. 3).

The initial dataset of maritime ports was acquired from the World Food Programme's Global Ports database, containing approximately 3 600 global ports, including river, lake, and minor ports. The final maritime network is in Fig. 4.

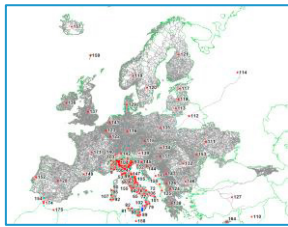


Fig. 2. Road network model

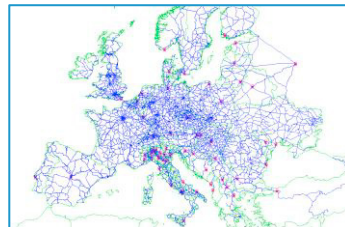


Fig. 3. Railway network model

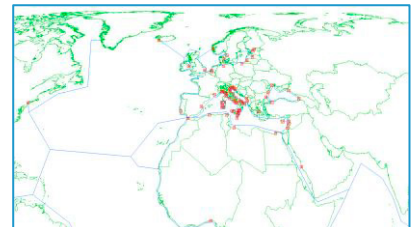


Fig. 4. Maritime network model

Multimodal routes are based on the following criteria. For road transport, the shortest route is chosen. For the road and rail chain, access and egress by road to and from the nearest railway terminal is considered. The connection between origin and destination railway terminal is direct and based on shortest route. For the road and maritime chain, access and egress by road to and from the nearest port is considered. The connection between origin and destination port is direct and based on shortest route.

2.7. Transport data equations

Transport data from Eurostat are classified according to the NST2007 system for road and rail modes. They are aggregated into the three sectors (dry bulk, liquid bulk, other goods) based on the correspondence table NST/R-CN2008-CPA2008-NST2007 in conjunction with the SAMGODS model. For maritime, Eurostat data are already classified according to the three sectors.

After processing, the following origin-destination flows in weight (tonnes/year) by mode chain are available for each sector:

- road traffic from Italy to other countries, $S_{j,road}, j \in J_2$, and from other countries to Italy, $S_{i,road}, i \in J_2$;
- total rail traffic involving Italy as origin and/or destination, S_{rail} ;
- maritime traffic from Italian ports to Italian ports, S_{mar} ;
- maritime traffic from Italian ports to ports of other countries, $S_{j,mar}, j \in J_2$, and from ports of other countries to Italian ports, $S_{i,mar}, i \in J_2$.

Then, the following equations hold:

$$\sum_{i \in J_1} X_{ij} P_{ij,road} \left(\frac{1}{p_i \tau_{ij}} \right) = S_{j,road}, j \in J_2 \quad (11)$$

$$\sum_{j \in J_1} X_{ij} P_{ij,road} \left(\frac{1}{p_i \tau_{ij}} \right) = S_{i,road}, i \in J_2 \quad (12)$$

$$\sum_{i \in J_1} \sum_{j \in J_1/i} X_{ij} P_{ij,rail} \left(\frac{1}{p_i \tau_{ij}} \right) + \sum_{i \in J_1} \sum_{j \in J_2} X_{ij} P_{ij,rail} \left(\frac{1}{p_i \tau_{ij}} \right) + \sum_{i \in J_2} \sum_{j \in J_1} X_{ij} P_{ij,rail} \left(\frac{1}{p_i \tau_{ij}} \right) = S_{rail} \quad (13)$$

$$\sum_{i \in J_1} \sum_{j \in J_1/i} X_{ij} P_{ij,mar} \left(\frac{1}{p_i \tau_{ij}} \right) = S_{mar} \quad (14)$$

$$\sum_{i \in J_1} X_{ij} P_{ij,mar} \left(\frac{1}{p_i \tau_{ij}} \right) = S_{j,mar}, j \in J_2 \quad (15)$$

$$\sum_{j \in J_1} X_{ij} P_{ij,mar} \left(\frac{1}{p_i \tau_{ij}} \right) = S_{i,mar}, i \in J_2 \quad (16)$$

where p_i are (factory-gate) price indices specific of production zone, which are needed to convert monetary flows into weight flows.

For the other goods sector, there are distinct maritime data related to RoRo, container and non-RoRo non-container, which add Eqs to Eqs (14), (15) and (16). Air transport also adds Eqs.

3. Application

3.1. Calibration

Consider a base year with available transport data as in section 2.7, as well as travel time and monetary costs by mode chain and by production-consumption pair. Let trade flows X_{ij} divided by the product of the associated price index specific of production zone p_i times the trade cost markup τ_{ij} be expressed by $\delta_i \gamma_j (\tau_{ij})^{-\sigma}$, where δ_i and γ_j are estimation constants specific of, respectively, production zones and consumption zones. For the transport cost markup, we assume the empirical specification $\tau_{ij} = a + b c_{ij}$.

With these expressions, a least squares estimation problem is formulated, where each residual is the difference between the modelled flows and the r.h.s. of transport data Eqs (11) to (16). Solution provides constants δ_i and γ_j of production and consumption zones, elasticity of substitution σ , coefficients a and b , and alternative specific constants ASC_n . The coefficients β_t and β_c are from Jensen et al. (2019).

Initial computations for dry bulk in 2019 have confirmed the validity of the approach. The code is written in Python. To solve the least squares problem, the GEKKO optimization library is used. The algorithm is an interior point line search filter method (Wächter and Biegler, 2006). With Italy modelled as single zone with travel times set at the mean across provinces, solution is obtained in less than 5 seconds with a 2.70 GHz CPU, 16 GB RAM. The estimates are: $\sigma = 1.432$, $a = 3.189$, $b = 1.002$, $ASC_{road} = 0$, $ASC_{rail} = -3.984$, $ASC_{mar} = 4.020$.

3.2. Counterfactual scenario analysis

Consider the base year, i.e. the one of the performed calibration, and one or more counterfactual transport scenarios, modelled by changing bilateral trade cost markup. Country-specific price indices in the base year can be obtained by equating modelled flows and Eurostat transport data. Equilibrium analysis includes two comparative statics cases. The one of conditional general equilibrium, with exogenous (at base year) price indices and import and export values. And the one of full general equilibrium, with endogenous factory-gate prices responding to changing transport costs and translating into changes in import and export.

4. Conclusions

A novel framework aimed to support short-term transport policy making has been presented. Policies are not restricted to new infrastructure, but extend to any intervention on transport supply that modifies the generalized cost perceived by the users. Research to complete calibrations is ongoing. Future research will apply equilibrium analysis to counterfactual transport scenarios.

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